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COMPUTING OF SOME DEGREE BASED TOPOLOGICAL INDICES OF $K_M \odot K_N$

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ABSTRACT

In this paper, we study some degree-based topological indices, such as, first Zagreb index ($M_1(G)$), second Zagreb index ($M_2(G)$), third Zagreb index ($M_3(G)$), Hyper-Zagreb index ($HM(G)$), Sigma index ($\sigma(G)$), Sombor index ($SO(G)$), Forgotten index ($F(G)$), SK index, SK_1 index and SK_2 index of $K_m \odot K_n$

MSC: 05C07, 05C10.

KEYWORDS: *Zagreb Indices, Hyper-Zagreb index, Sigma index, Sombor index, Forgotten index, SK index, SK1 index, SK2 index and Corona product.*

INTRODUCTION

The study of graphs, which are mathematical structures made up of vertices (also known as nodes) and the edges (also known as arcs) that link these vertices, is the focus of the field of graph theory. When utilizing graphs to explore molecule structures and their characteristics, chemical graph theory is an essential tool. It entails representing molecules as graphs, with chemical bonds acting as edges and atoms acting as vertices. In this study, we consider only simple, finite, undirected and connected graphs.

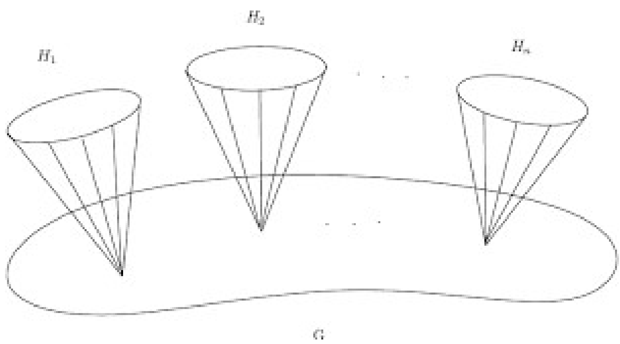

Figure 1: Corona Product of two graphs G and H

A graph G is an ordered triple ($V(G)$, $E(G)$, $\chi_{(G)}$) consisting of a nonempty set $V(G)$ of vertices, a set $E(G)$, disjoint from $V(G)$, of edges, and an incidence function $\chi_{(G)}$ that associates with each edge of G an unordered pair of (not necessarily distinct) vertices of G. If e is an edge and u, v are vertices such that $\chi_G(e) = uv$, then e is an edge that joins u and v, the vertices u and v are called the ends of e. The degree of a vertex v in a graph G, denoted by $d_G(v)$, is the number of edges of G incident with v, each loop counting as two edges. A complete graph is a simple graph in which any two vertices are adjacent.

Definition 1.1. *[11] The corona product of two graphs G and H is defined as the graph obtained by taking one copy of G and* $|V(G)|$ *copies of H and joining the* i^{th} *vertex of G to every vertex in the* i^{th} *copy of H.*

A topological graph index, also known as a molecular descriptor, is a mathematical formula that may be applied to any network representing a molecular structure. This index allows you to analyze mathematical quantities and study some of the molecule's physical features. As a result, it provides a cost-effective and time-saving alternative to laboratory trials.

In [8], I. Gutman and N. Trinajstic introduced the first $M_1(G)$ and second $M_2(G)$ Zagreb indexes, which were described as

$M_1(G) = \Sigma_{v \epsilon V(G)} d^2(v)$,

$M_2(G) = \Sigma_{u,v \epsilon E(G)}(d_G(u) d_G(v))$.

In [3], G. H. Fath-Tabar described the third Zagreb index $M_2(G)$

as

$M_3(G) = \Sigma_{u,v \epsilon E(G)} |d_G(u) - d_G(v)|$.

In [4], G. H. Shirdel, H. Rezapour, and A. M. Sayadi defined Hyper

Zagreb index HM (G) as

$HM(G) = \Sigma_{u,v \epsilon E(v)}(d_G(u) + d_G(v))^2$.

In [7], I Gutman, M. Togan, A. Yurttas, A. S. Cevik, and I. N.

Cangul defined the sigma index as

$\sigma(G) = \Sigma_{u,v \epsilon E(G)}(d_G(u) - d_G(v))^2$.

In [6], I. Gutman presented a unique degree-based topological index known as the Sombor index. It was motivated by the geometric interpretation of the degree radii of the edges and defined as

$SO(G) = \Sigma_{u,v \epsilon E(G)} \sqrt{(d(u)^2 + d(v)^2}$

In [2] , I. Gutman and B. Furtula defined the forgotten topological index (F-index) as follows:

$F(G) = \Sigma_{v \epsilon V(G)} d^3(u)$.

In [12], V. S. Shegahalli and R. Kanabur introduced the SK, SK_1

and SK_2 index as follows:

$$SK(G) = \frac{1}{2}\sum_{u,v\in E(G)}(d(u) + d(v)),$$

$$SK_1(G) = \frac{1}{2}\sum_{u,v\in E(G)}(d(u)\,d(v)),$$

$$SK_2(G) = \frac{1}{4}\sum_{u,v\in E(G)}(d(u) + d(v))^2.$$

MAIN RESULTS

In this section, we compute some degree-based topological indices of corona product of $K_m \odot K_n$

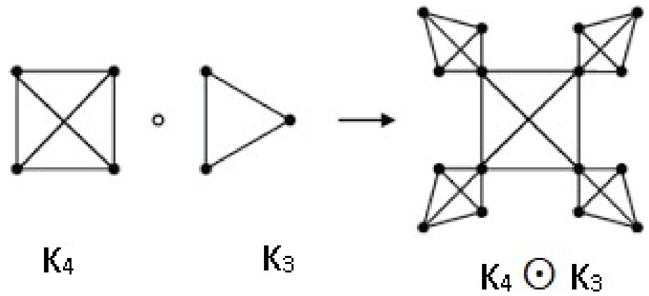

Figure 2: Corona Product of $K_4 \odot K_3$

Definition 2.1. The corona product of two complete graphs K_m and K_n is defined as the graph obtained by taking one copy of K_m and m copies of K_n and joining the i^{th} vertex of K_m to every vertex in the i^{th} copy of K_n. From Figure 2, we have $|V(K_m \odot K_n)| = m(n+1)$ and $|E(K_m \odot K_n)| = \frac{m(m-1)}{2} + \frac{mn(n-1)}{2}$

Theorem 2.2. Let K_m and K_n be two complete graphs with $m, n \geq 1$. Then

(i) $M_1(K_m \odot K_n) = m[(m-1)^2 + n^2 + 2n(m-1) + n^3]$.

(ii) $M_2(K_m \odot K_n) = \frac{m(m-1)}{2}[m-1+n]^2 + \frac{mn^2}{2}(n^2 + n + 2m - 2)$.

(iii) $M_3(K_m \odot K_n) = mn(m-1)$.

Proof. Let K_m and K_n be two complete graphs with $m, n \geq 1$.

(i) By using the definition of the first Zagreb index, we obtain

$$\begin{aligned} M_1(K_m \odot K_n) &= \sum_{v\in V(K_m \odot K_n)} d^2(v), \\ &= m[(m-1)^2 + n^2 + 2n(m-1)] + mn[(n-1)+1]^2, \\ &= m[(m-1)^2 + n^2 + 2n(m-1)] + mn(n)^2, \\ &= m[(m-1)^2 + n^2 + 2n(m-1)] + mn^3, \\ &= m[(m-1)^2 + n^2 + 2n(m-1) + n^3]. \end{aligned}$$

(ii) By using the definition of the second Zagreb index, we obtain

$$
\begin{aligned}
M_2(K_m \odot K_n) &= \sum_{u,v\in E(K_m \odot K_n)} (d(u)d(v)), \\
&= \frac{m(m-1)}{2}[((m-1)+n)((m-1)+n)] \\
&+ \frac{mn(n-1)}{2}[((n-1)+1)((n-1)+1)] \\
&+ mn[(m-1+n)((n+1)-1)], \\
&= \frac{m(m-1)}{2}[(m-1+n)]^2 + \frac{mn(n-1)}{2}(n^2) \\
&+ mn[(m-1+n)(n)], \\
&= \frac{m(m-1)}{2}[(m-1+n)]^2 + mn^2\left[\frac{n(n-1)}{2} + (m+n-1)\right], \\
&= \frac{m(m-1)}{2}[(m-1+n)]^2 + \frac{mn^2}{2}[n^2+n+2m-2].
\end{aligned}
$$

(iii) Using the definition of the third Zagreb index, we obtain

$$
\begin{aligned}
M_3(K_m \odot K_n) &= \sum_{u,v\in E(K_m \odot K_n)} |d(u)-d(v)|, \\
&= \frac{m(m-1)}{2}|((m-1)+n)-((m-1)+n)| \\
&+ \frac{mn(n-1)}{2}|((n-1)+1)-((n-1)+1)| \\
&+ mn|(m-1+n)-((n+1)-1)|, \\
&= mn|m-1+n-n+1-1|, \\
&= mn(m-1).
\end{aligned}
$$

Theorem 2.3. Let K_m and K_n be two complete graphs with $m, n \geq 1$. Then

(i) $HM(K_m \odot K_n) = 2m(m-1)[(m-1)+n]^2 + 2mn^3(n-1) + mn[2n+(m-1)]^2$

(ii) $\sigma(K_m \odot K_n) = mn(m-1)^2$.

Proof. Let K_m and K_n be two complete graphs with $m, n \geq 1$.

(i) By using the definition of the Hyper - Zagreb index, we obtain

$$
\begin{aligned}
HM(K_m \odot K_n) &= \sum_{u,v\in E(K_m \odot K_n)} (d(u)+d(v))^2 \\
&= \frac{m(m-1)}{2}[((m-1)+n)+((m-1)+n)]^2 \\
&+ \frac{mn(n-1)}{2}[((n-1)+1)+((n-1)+1)]^2 \\
&+ mn[(m-1+n)+((n-1)+1)]^2, \\
&= \frac{m(m-1)}{2}[2(m-1+n)]^2 \\
&+ \frac{mn(n-1)}{2}[2n]^2 + mn[2n+m-1]^2, \\
&= 2m(m-1)[m-1+n]^2 + 2mn^3[n-1] \\
&+ mn[2n+m-1]^2.
\end{aligned}
$$

(ii) By using the definition of the Sigma index, we obtain

$$\sigma(K_m \odot K_n) = \sum_{u,v \in E(K_m \odot K_n)} \left(d(u) - d(v)\right)^2,$$

$$= \frac{m(m-1)}{2}\left[\left((m-1)+n\right) - \left((m-1)+n\right)\right]^2$$

$$+\frac{mn(n-1)}{2}\left[\left((n-1)+1\right) - \left((n-1)+1\right)\right]^2$$

$$+mn\left[(m-1+n) - \left((n-1)+1\right)\right]^2,$$

$$= mn[m-1+n-n+1-1]^2,$$

$$= mn(m-1)^2.$$

Theorem 2.4. Let K_m and K_n be two complete graphs with $m, n \geq 1$. Then

(i) $$SO(K_m \odot K_n) = \frac{m(m-1)}{2}\left[\sqrt{2\left((m-1)+n\right)^2}\right] + \frac{mn(n-1)}{2}\left[\sqrt{2n^2}\right] + mn\left[\sqrt{(m-1+n)^2+n^2}\right].$$

(ii) $F(K_m \odot K_n) = m(m-1)^3 + mn[n(n^2+n-3)] + m(3m-6) + 3(nm+1)$

Proof. Let K_m and K_n be two complete graphs with $m, n \geq 1$.

(i) By using the definition of the Sombor index, we obtain

$$SO(K_m \odot K_n) = \sum_{u,v \in E(K_m \odot K_n)} \sqrt{(d(u)^2 + d(v)^2)},$$

$$= \frac{m(m-1)}{2}\left[\sqrt{(m-1+n)^2 + (m-1+n)^2}\right]$$

$$+\frac{mn(n-1)}{2}\left[\sqrt{(n-1+1)^2 + \left((n-1)+1\right)^2}\right]$$

$$+mn\left[\sqrt{\left((m-1)+n\right)^2 + \left((n-1)+1\right)^2}\right],$$

$$= \frac{m(m-1)}{2}\left[\sqrt{2\left((m-1)+n\right)^2}\right]$$

$$+\frac{mn(n-1)}{2}\left[\sqrt{2n^2}\right] + mn\left[\sqrt{(m-1+n)^2+n^2}\right].$$

(ii) By using the definition of the F- Index, we obtain

$$F(K_m \odot K_n) = \Sigma_{v \in V(K_m \odot K_n)} d^3(v)$$

$$= m[(m-1)+n]^3 + mn[(n-1)+1]^3$$

$$= m[(m-1)+n]^3 + mn(n)^3$$

$$= m[(m-1)^3 + 3(m-1)^2 n + 3(m-1)n^2 + n^3] + mn^4$$

$$= m(m-1)^3 + mn[n(n^2+n-3)] + m(3m-6) + 3(nm+1).$$

Theorem 2.5. Let K_m and K_n be two complete graphs with $m, n \geq 1$. Then

(i) $SK(K_m \odot K_n) = \frac{1}{2}[m(m-1)(m+n-1) + mn(n^2+n+m-1)]$,

(ii) $SK_1(K_m \odot K_n) = \frac{1}{4}[m(m-1)(m-1+n)^2 + mn^2(n^2+n+2m-2)]$,

$$SK_2(K_m \odot K_n) = \frac{1}{4}[2m(m-1)(m-1+n)^2 + 2mn^3(n-1) + mn(2n+m-1)^2].$$

Proof. Let K_m and K_n be two complete graphs with $m, n \geq 1$.

(i) By using the definition of the SK index, we obtain

$$SK(K_m \odot K_n) = \frac{1}{2}\sum_{u,v \in E(K_m \odot K_n)} (d(u) + d(v)),$$

$$= \frac{1}{2}\left[\begin{array}{c} \frac{m(m-1)}{2}[((m-1)+n) + ((m-1)+n)] \\ + \frac{mn(n-1)}{2}[((n-1)+1) + ((n-1)+1)] \\ + mn[(m-1+n) + ((n-1)+1)] \end{array}\right].$$

$$= \frac{1}{2}\left[\frac{m(m-1)}{2}(2m+2n-2) + \frac{mn(n-1)}{2}(2n) + mn(m+2n-1)\right].$$

$$= \frac{1}{2}[m(m-1)(m+n-1) + mn(n^2+n+m-1)].$$

(ii) By using the definition of the SK_1 index, we obtain

$$SK_1(K_m \odot K_n) = \frac{1}{2}\sum_{u,v \in E(K_m \odot K_n)} (d(u)d(v)),$$

$$= \frac{1}{2}\left[\begin{array}{c} \frac{m(m-1)}{2}[((m-1)+n)((m-1)+n)] \\ + \frac{mn(n-1)}{2}[((n-1)+1)((n-1)+1)] \\ + mn[(m-1+n)((n-1)+1)] \end{array}\right],$$

$$= \frac{1}{2}\left[\begin{array}{c} \frac{m(m-1)}{2}[(m-1+n)]^2 + \frac{mn(n-1)}{2}(n^2) \\ + mn[(m-1+n)(n)] \end{array}\right],$$

$$= \frac{1}{4}[m(m-1)(m-1+n)^2 + mn^2(n^2+n+2m-2)].$$

(iii) By using the definition of the SK_2 index, we obtain

$$SK_2(K_m \odot K_n) = \frac{1}{4}\sum_{u,v\in E(K_m \odot K_n)} (d(u)+d(v))^2$$

$$= \frac{1}{4}\left[\begin{array}{c} \frac{m(m-1)}{2}[((m-1)+n)+((m-1)+n)]^2 \\ +\frac{mn(n-1)}{2}[((n-1)+1)+((n-1)+1)]^2 \\ +mn[(m-1+n)+((n-1)+1)]^2 \end{array}\right],$$

$$= \frac{1}{4}\left[\begin{array}{c} \frac{m(m-1)}{2}[2(m-1+n)]^2 + \frac{mn(n-1)}{2}[2n]^2 \\ +mn[2n+m-1]^2 \end{array}\right]$$

$$= \frac{1}{4}\left[\begin{array}{c} 2m(m-1)[m-1+n]^2 + 2mn^3[n-1] \\ +mn[2n+m-1]^2 \end{array}\right].$$

CONCLUSION

In this research, we examined the results of some degree-based topological indices for the corona product of two complete graphs. Additional topological indices for this graph can be constructed for future exploration.

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